Incoherent Estimation on Co-Channel Interference Probability for Microcellular Systems

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Abstract—**Recently, a paper [1] offered an approximate analysis of the co-channel interference (CCI) probability for a microcellular system that has multiple shadowed Rayleigh interfering signals and a shadowed Rician desired signal. In that analysis, coherent addition of summing multiple interfering signals was assumed. However, it has been shown that incoherent addition is a more realistic assumption [2]. In this paper, we present two approaches—approximation and exact analysis—for calculating the CCI probability under incoherent addition. The approximate analysis is extended from the macrocellular case [2]. The exact analysis, newly developed in this paper, is then used to check the accuracy of the approximation. It is shown that the results of the incoherent approximate analysis differ by an amount of 20~30%. In addition, we explore the effect of correlation between the shadowing interferers. We find that in a shadowed-Rician/shadowed-Rayleigh channel, the effect of this kind of correlation increases the CCI probability for cases having a small spread in shadowing and large reuse distance. But if the spread in shadowing is large, the result can be just the opposite—this correlation yields a better performance. It is also found that in a shadowed-Rayleigh (desired)/shadowed-Rayleigh (interfering) channel, regardless of degree of spread in shadowing, this correlation seems always to improve the performance.**

I. Introduction

The CO-CHANNEL INTERFERENCE (CCI) probability is an important performance measure for cellular mobile systems. The CCI probability is the probability of instantaneous signal-to-interference (S/I) ratio below a required threshold. It is affected by both the desired signal and the interference model. In microcellular mobile systems, the desired signal model consists of three components: 1) a fast varying component with a Rician distribution, 2) a slowly varying shadowing component, and 3) a deterministic path loss for area mean power. The model of a single interferer is similar to the desired signal model described above except that the fast varying part is replaced with a Rayleigh distributed random variable. The difficult part, however, is to develop the interference model of multiple interferer added together.

Two kinds of models have been proposed to model the addition of multiple interferer. In [3], the multiple interfering signals are assumed to add coherently, i.e., with the same phase. We will call the CCI probability of this kind the *coherent* CCI probability. On the other hand, the CCI probability under an incoherent addition of interfering signals [2] is called the *incoherent* CCI probability. It has been mentioned in [2] and [4] that the incoherent assumption is more reasonable, especially when the number of interfering signals is large.

For a cellular environment with a shadowed-Rician/shadowed-Rayleigh propagation model, Prasad and Kegel's paper [1] analyzed only the *coherent* CCI probability. In this paper, we show two approaches—approximation and exact analysis—for calculating the *incoherent* CCI probability. The approximate analysis, requiring less computation time, is extended from the results in macrocellular systems [2] to the microcellular environment. The exact analysis is newly developed here. We shall compare the difference between the two. In addition, we study the effect on the CCI probability of the correlation between the shadowing components of the interferers. We consider a strongly correlated case with correlation coefficient which is equal to one.

The remaining parts of this paper are organized as follows. For comparison, Section II first reviews the results of coherent approximation of the CCI probability, which are taken from Prasad and Kegel's paper [1]. An incoherent approximation of the CCI probability in the macrocellular system is then extended to the microcellular systems. In Section III, the exact analysis of the CCI probability for the uncorrelated shadowing and correlated shadowing is presented. In Section IV are given to compare the CCI probability as well as spectrum efficiency. We conclude the discussion in Section V.

II. Approximate Analysis of CCI Probability

A. Previous Results: Coherent Case

The results from [1] for the coherent CCI probability are summarized in this section. Similar notations are adopted here. The fast varying component of the desired signal strength in microcellular systems, as mentioned in the previous section, is usually modeled by a Rician distributed random variable. Given the local mean p_{od}, the probability density function (pdf) of the instantaneous signal power p_d is described as

$$f_{p_d}(p_d \mid p_{od}) = \frac{R_d+1}{P_{od}} \exp\left[-\frac{(R_d+1)(2p_d+s^2)}{2p_{od}}\right] \times I_0\left(\frac{\sqrt{2p_d}s(R_d+1)}{p_{od}}\right) \quad (1)$$

where R_d is the Rician factor, s is the peak value of line-of-sight component, and $I_0()$ is the zeroth-order modified Bessel function of the first kind.

The slowly varying local mean p_{od} represents the shadow effect caused by the terrain and buildings, etc. The pdf of

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local mean p_{od} is given by

$$f_{p_{od}}(p_{od}) = \frac{1}{\sqrt{2\pi}\sigma_d p_{od}} \exp\left[-\frac{(\ln p_{od} - m_d)^2}{2\sigma_d^2}\right] \quad (2)$$

where σ_d is the spread of shadowing and m_d is the logarithm of the area mean ϵ_d, i.e., $m_d = \ln \epsilon_d$. The path loss model for area mean, ϵ_d, will be given in Section IV.

The composite pdf of the instantaneous desired signal power with shadow effect and Rician fading is thus described as

$$\begin{aligned} f_{p_d}(p_d) &= \int_0^\infty \frac{R_d+1}{\sqrt{2\pi}\sigma_d p_{od}^2} \\ &\times \exp\left[-\frac{(R_d+1)(2p_d+s^2)}{2p_{od}} - \frac{(\ln p_{od} - m_d)^2}{2\sigma_d^2}\right] \\ &\times I_0\left(\frac{\sqrt{2p_d}s(R_d+1)}{p_{od}}\right) dp_{od}. \end{aligned} \quad (3)$$

Equation (3) is the same as (9) in [1].

The interfering signal, denoted by p_i, generally contains no line-of-sight component, i.e. $R_d = 0$ and $s = 0$ in (3). Thus the pdf of p_i is described as a Rayleigh distribution superimposed on a log-normal distributed random variable, which is

$$\begin{aligned} f_{p_i}(p_i) = \int_0^\infty & \frac{1}{\sqrt{2\pi}\sigma_i p_{oi}^2} \\ & \times \exp\left[-\frac{p_i}{p_{oi}} - \frac{(\ln p_{oi} - m_i)^2}{2\sigma_i^2}\right] dp_{oi} \end{aligned} \quad (4)$$

where p_{oi} is the local mean, m_i is the logarithm of the area mean ϵ_i, and σ_i is the spread of shadowing.

Assume n uncorrelated multiple shadowed Rayleigh interfering signals add coherently and let $p_n = \sum_{i=1}^n p_i$. Using an approximation technique of [5], Prasad and Kegel's paper [1] approximated the pdf of p_n as

$$\begin{aligned} f_{p_n}(p_n) &= \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_n p_{on}^2} \cdot \exp\left[-\frac{p_n}{p_{on}} - \frac{(\ln p_{on} - m_n)^2}{2\sigma_n^2}\right] dp_{on} \end{aligned} \quad (5)$$

where $p_{on} = \sum_{i=1}^n p_{oi}$, $\epsilon_n = e^{m_n}$ is the composite area mean power, and σ_n is the spread of shadowing. With the pdf of the desired signal and interference given in (3) and (5), the coherent CCI probability for the shadowed-Rician/shadowed-Rayleigh propagation model is given by

$$\begin{aligned} F_{\text{coh}}(CI \mid n) &\triangleq \text{Prob}[p_d/p_n < \lambda_{\text{th}}] \\ &= \int_0^\infty \int_0^\infty \frac{W}{2\pi\sigma_d\sigma_n p_{od} p_{on}} \\ &\cdot \exp\left[-\frac{(\ln p_{od} - m_d)^2}{2\sigma_d^2}\right] \cdot \exp\left[-\frac{(\ln p_n - m_n)^2}{2\sigma_n^2}\right] \\ &\cdot \exp\left[-\frac{R_d W p_{od}}{\lambda_{\text{th}} p_{on}(R_d+1)}\right] dp_{on} dp_{od} \end{aligned} \quad (6)$$

where W is defined as [1, (12)]

$$W = \frac{\lambda_{\text{th}}}{\frac{p_{od}}{p_{on}(R_d+1)} + \lambda_{\text{th}}} \quad (7)$$

and λ_{th} is the required threshold at the receiver.

B. Incoherent Case

The results summarized above are for coherent addition of interfering signals. However, it has been mentioned in [2] and [4] that incoherent addition of multiple interfering signals is a more reasonable assumption, especially when the number of interfering signals is large. We will follow a similar approximation technique as in [2] and analyze the incoherent case below. The exact analysis will be given in the next section. The incoherent CCI probability for the shadowed-Rician/shadowed-Rayleigh propagation model will be be expressed in Hermite integration form, which is much easier to calculate by using the formula presented in [4].

With incoherent summation, the composite pdf for the multiple shadowed Rayleigh co-channel interfering signals p_n is shown to be [1], [2]

$$f_n(p_n) = \frac{1}{\sqrt{2\pi}\sigma_n p_n} \exp\left[-\frac{(\ln p_n - m_n)^2}{2\sigma_n^2}\right] \quad (8)$$

where $p_n = \sum_{i=1}^n p'_{oi}$. Note that p'_{oi} is a log-normal distributed random variable that closely approximates the composite pdf of a Rayleigh fading signal superimposed on a log-normal shadow fading component [6]. With (3) and (8), the incoherent CCI probability can be calculated as follows:

$$\begin{aligned} F_{i\text{coh}}(CI \mid n) &= \text{Prob}[p_d/p_n < \lambda_{\text{th}}] \\ &= \int_0^\infty \int_0^\infty \frac{1}{2\pi\sigma_d\sigma_n p_{od} p_n} \cdot \exp\left[-\frac{(\ln p_{od} - m_d)^2}{2\sigma_d^2}\right] \\ &\cdot \exp\left[-\frac{(\ln p_n - m_n)^2}{2\sigma_n^2}\right] \cdot \int_0^{\lambda_{\text{th}} p_n} \frac{R_d+1}{p_{od}} \\ &\cdot \exp\left[-\frac{(R_d+1)(2p_d+s^2)}{2p_{od}}\right] \\ &\times I_0\left(\frac{\sqrt{2p_d}s(R_d+1)}{p_{od}}\right) dp_d dp_n dp_{od} \end{aligned} \quad (9)$$

$$\begin{aligned} &= \int_0^\infty \int_0^\infty \frac{1}{2\pi\sigma_d\sigma_n p_{od} p_n} \\ &\cdot \exp\left[-\frac{(\ln p_{od} - m_d)^2}{2\sigma_d^2}\right] \cdot \exp\left[-\frac{(\ln p_n - m_n)^2}{2\sigma_n^2}\right] \\ &\cdot \left[1 - Q_1\left(s\sqrt{\frac{R_d+1}{p_{od}}}, \sqrt{2(R_d+1)\lambda_{\text{th}}\frac{p_n}{p_{od}}}\right)\right] dp_n dp_{od} \end{aligned} \quad (10)$$

where Q_1 is the Marcum Q function [7]. Note that integration over p_d in (9) is equivalent to calculating the cumulative density function (cdf) of a Rician distributed random variable [8].

By changing the variables $t_1 = (\ln p_{od} - m_d)/\sqrt{2}\sigma_d$ and $t_2 = (\ln p_n - m_n)/\sqrt{2}\sigma_n$, we will get

$$F_{i\text{coh}}(CI \mid n) = \int_0^\infty dt_1 \int_0^\infty \frac{1}{\pi} \cdot \exp[-t_1^2] \cdot \exp[-t_2^2] \times \left[1 - Q_1\left(\sqrt{2R_d}, \sqrt{\frac{2(R_d+1)\lambda_{\text{th}} \exp[\sqrt{2}\sigma_n t_2 + m_n]}{\exp[\sqrt{2}\sigma_d t_1 + m_d]}}\right)\right] dt_2. \tag{11}$$

Using transformation $u_1 = (\sigma_d t_2 + \sigma_n t_1)/\sqrt{\sigma_n^2 + \sigma_d^2}$ and $u_2 = (\sigma_n t_2 - \sigma_d u_1)/\sqrt{\sigma_n^2 + \sigma_d^2}$, and then integrating over u_1, (11) can be further simplified to the following Hermite integration form:

$$F_{i\text{coh}}(CI \mid n) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f_{i\text{coh}}(u_2) \exp\left(-u_2^2\right) du_2 \tag{12}$$

where

$$f_{i\text{coh}}(u_2) = 1 - Q_1\left(\sqrt{2R_d}, \sqrt{2(R_d+1)\lambda_{\text{th}} \frac{\epsilon_n}{\epsilon_d} \exp\left[-\sqrt{2(\sigma_d^2 + \sigma_n^2)}u_2\right]}\right) \tag{13}$$

$\epsilon_d = e^{m_d}$ is the desired area mean power, and $\epsilon_n = e^{m_n}$ is the interfering signal power.

We can apply the same technique to the *coherent* CCI probability in Section II-A to further simplify the results. The reasons for this further simplification are two-fold. First, we can apply Hermite polynomial approach [4] to get results for both cases quickly. Second, we can easily compare the results of microcellular systems with those of macrocellular systems in [2], [3], and [9]. Following the deriving procedures from (10)–(13), we can replace the double integral in (6) with a similar Hermite integration form shown below

$$F_{\text{coh}}(CI \mid n) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f_{\text{coh}}(u_2) \exp\left(-u_2^2\right) du_2 \tag{14}$$

where

$$f_{\text{coh}}(u_2) = \frac{\exp\left(-\frac{R_d H(u_2)}{1+H(u_2)}\right)}{1 + H(u_2)} \tag{15}$$

and

$$H(u_2) = \frac{1}{(R_d+1)\lambda_{\text{th}}} \left(\frac{\epsilon_d}{\epsilon_n}\right) \exp\left(-\sqrt{2(\sigma_d^2 + \sigma_n^2)}u_2\right). \tag{16}$$

Here $\epsilon_d = e^{m_d}$ and $\epsilon_n = e^{m_n}$ are the desired and the interfering signal power, respectively. By letting $R_d = 0$ in (13) and (15), we will get (17) and (22) in [2]. Note that an error in (17) of [2] has been corrected in [9]. The corrected formula is the same as the special case $R_d = 0$ of (13).

III. Incoherent CCI Probability: Exact Analysis

The approximate analysis given in the previous section has a modest computation complexity, but how accurate is it? We will answer this question by offering an exact analysis in this section. Both correlated and uncorrelated log-normal shadowing are discussed here. Our first step is to express the conditional CCI probability as

$$F(CI \mid n) = \text{Prob}\left[\frac{p_d}{p_n} < \lambda_{\text{th}}\right] = \int_0^\infty \left[\int_{\frac{1}{\lambda_{\text{th}}}} f_{p_n}(zw)dz\right] f_{p_d}(w) w dw \tag{17}$$

where $f_{p_n}()$ and $f_{p_d}()$ are the pdfs of interfering signal power and desired signal power, respectively. The derivation is given in Appendix B.

A. Uncorrelated Log-Normal Shadowing Case

Consider the case of the uncorrelated log-normal shadowing, which is the case with the previous two sections. The joint pdf of the log-normal shadowing components is the product of the pdfs of each log-normal random variable p_{oi}, which is

$$f_{p_{o1}\cdots p_{on}}(p_{o1}, \ldots, p_{on}) = \prod_{i=1}^{n} f_{oi}(p_{oi}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_i p_{oi}} \exp\left[-\frac{(\ln p_{oi} - m_i)^2}{2\sigma_i^2}\right]. \tag{18}$$

For each shadowed Rayleigh interferer, given the local mean power p_{oi}, the individual interference power is characterized by an exponential random variable, of which the pdf is

$$f_{p_i}(p_i \mid p_{oi}) = \frac{1}{p_{oi}} \exp\left(\frac{-p_i}{p_{oi}}\right) \tag{19}$$

and the characteristic function is

$$\Psi_{p_i}(s \mid p_{oi}) = \mathcal{L}\{f_{p_i}(p_i \mid p_{oi}); s\} = \frac{1}{1 + s p_{oi}} \tag{20}$$

where $\mathcal{L}\{\}$ represents the Laplace transform. Thus the exact expression for the joint interference power with incoherent addition can be found by convoluting the n pdfs of the individual interference power. Since each instantaneous signal power can be assumed independent in a Rayleigh fading environment, the characteristic function of the joint interference power with incoherent addition is the multiplication of the characteristic function of each individual interferer; that is

$$\Psi_{p_n}(s \mid p_{o1}, \ldots, p_{on}) = \prod_{i=1}^{n} \Psi_{p_i}(s \mid p_{oi}) = \prod_{i=1}^{n} \frac{1}{1 + s p_{oi}}. \tag{21}$$

The inverse Laplace transformation of (21) can be expressed by

$$f_{p_n}(p_n \mid p_{o1}, \ldots, p_{on}) = \mathcal{L}^{-1}\{\Psi_{p_n}(s \mid p_{o1}, \ldots, p_{on})\} = \sum_{i=1}^{n} \frac{p_{oi}^{n-2} \exp\left(\frac{-p_n}{p_{oi}}\right)}{\prod_{j=1, j\neq i}^{n} (p_{oi} - p_{oj})}. \tag{22}$$

It should be noted that we assume $p_{oi} \neq p_{oj}$ in the above equation. This is fine since the case $p_{oi} = p_{oj}$ is only a point in the joint density function of the interferers and has no probability mass. In the next section, we consider a channel with strongly correlated interferers where the case $p_{oi} = p_{oj}$ can't be ignored. We now substitute (22) into (17) and obtain

$$\begin{aligned} F(CI \mid n, p_{od}, p_{o1} \ldots, p_{on}) &= \int_0^\infty \left[\int_{\frac{1}{\lambda_{\text{th}}}} f_{p_n}(zw \mid p_{o1}, \ldots, p_{on}) dz \right] f_{p_d}(w \mid p_{od}) w dw \\ &= \int_0^\infty \sum_{i=1}^{n} \frac{p_{oi}^{n-1} \exp\left(\frac{-w}{p_{oi}\lambda_{\text{th}}}\right)}{\prod_{j=1, j\neq i}^{n} (p_{oi} - p_{oj})} f_{p_d}(w \mid p_{od}) dw \\ &= \sum_{i=1}^{n} \frac{p_{oi}^{n-1}}{\prod_{j=1, j\neq i}^{n} (p_{oi} - p_{oj})} \int_0^\infty e^{-\frac{w}{p_{oi}\lambda_{\text{th}}}} f_{p_d}(w \mid p_{od}) dw. \end{aligned} \tag{23}$$

The integration part in (23) is equivalent to the characteristic function of the desired signal power at the point ($\phi = 1/\lambda_{\text{th}} p_{oi}$). In Appendix A, the characteristic function of the pdf for the desired signal power (a noncentral Chi square random variable with two degree of freedom) is derived. By changing variable ($\phi = 1/\lambda_{\text{th}} p_{oi}$) in (A6), it can be written as

$$\begin{aligned} \int_0^\infty & e^{-\frac{w}{p_{oi}\lambda_{\text{th}}}} f_{p_d}(w \mid p_{od}) dw \\ &= \mathcal{L}\left\{ f_{p_d}(w \mid p_{od}); \frac{1}{p_{oi}\lambda_{\text{th}}} \right\} \\ &= \frac{R_d + 1}{R_d + 1 + \frac{p_{od}}{p_{oi}\lambda_{\text{th}}}} \exp\left[\frac{-R_d \frac{p_{od}}{\lambda_{\text{th}} p_{oi}}}{R_d + 1 + \frac{p_{od}}{p_{oi}\lambda_{\text{th}}}} \right]. \end{aligned} \tag{24}$$

Combining (23) and (24), we obtain

$$\begin{aligned} F_{\text{exact, ind}}&(CI \mid n, p_{od}, p_{o1}, \ldots, p_{on}) \\ &= \sum_{i=1}^{n} \frac{p_{oi}^{n-1}}{\prod_{j=1, j\neq i}^{n} (p_{oi} - p_{oj})} \frac{R_d + 1}{R_d + 1 + \frac{p_{od}}{p_{oi}\lambda_{\text{th}}}} \\ &\quad \times \exp\left[\frac{-R_d \frac{p_{od}}{\lambda_{\text{th}} p_{oi}}}{R_d + 1 + \frac{p_{od}}{p_{oi}\lambda_{\text{th}}}} \right]. \end{aligned} \tag{25}$$

We develop it further by averaging over $p_{od}, p_{o1}, \ldots, p_{on}$

$$\begin{aligned} F_{\text{exact, ind}}(CI \mid n) &= \int_0^\infty \cdots \int_0^\infty F_{\text{exact, ind}}(CI \mid n, p_{od}, p_{o1}, \ldots, p_{on}) \\ &\quad \times f_{p_{od}}(p_{od}) f_{p_{o1} \cdots p_{on}}(p_{o1} \cdots p_{on}) dp_{o1} \cdots dp_{on} \\ &= \int_0^\infty \cdots \int_0^\infty F_{\text{exact, ind}}(CI \mid n, p_{od}, p_{o1}, \ldots, p_{on}) \\ &\quad \times \frac{1}{\sqrt{2\pi}\sigma_d p_{od}} \exp\left[\frac{-(\ln p_{od} - m_d)^2}{2\sigma_d^2} \right] \\ &\quad \times \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_i p_{oi}} \exp\left[\frac{-(\ln p_{oi} - m_i)^2}{2\sigma_i^2} \right] dp_{od} dp_{o1} \cdots dp_{on}. \end{aligned} \tag{26}$$

We then change the variables $p_{od} = \exp\left(\sqrt{2} y_d \sigma_d + m_d\right)$, $p_{oi} = \exp\left(\sqrt{2} y_i \sigma_i + m_i\right)$, and thus obtain $F_{\text{exact, ind}}$ in the following Hermite integration form:

$$\begin{aligned} F_{\text{exact, ind}}&(CI \mid n) \\ &= \int_0^\infty \cdots \int_0^\infty \frac{G(y_d, y_1, \ldots, y_n)}{\sqrt{\pi}^{n+1}} \\ &\quad \times \exp[-(y_d^2 + y_1^2 + \cdots + y_n^2)] dy_d dy_1 \cdots dy_n \end{aligned} \tag{27}$$

where

$$\begin{aligned} G&(y_d, y_1, \ldots, y_n) \\ &= F_{\text{exact, ind}}(CI \mid n, p_{od}, p_{o1}, \ldots, p_{on})\big|_{p_{oi} = e^{\sqrt{2} y_i \sigma_i + m_i}; i=0,\ldots,n} \\ &= \sum_{i=1}^{n} \frac{1}{\prod_{j=1, j\neq i}^{n} (1 - \exp[\sqrt{2}(y_j\sigma_j - y_i\sigma_i) + m_j - m_i])} \\ &\quad \times \frac{R_d + 1}{R_d + 1 + \frac{1}{\lambda_{\text{th}}} \exp\left[\sqrt{2}(y_d\sigma_d - y_i\sigma_i) + m_d - m_i\right]} \\ &\quad \times \exp\left[\frac{-\frac{R_d}{\lambda_{\text{th}}} \exp\left[\sqrt{2}(y_d\sigma_d - y_i\sigma_i) + m_d - m_i\right]}{R_d + 1 + \frac{1}{\lambda_{\text{th}}} \exp\left[\sqrt{2}(y_d\sigma_d - y_i\sigma_i) + m_d - m_i\right]} \right]. \end{aligned} \tag{28}$$

B. Strongly Correlated Log-Normal Shadowing

To learn the effect of correlation between the shadowing components of the interferers on the CCI probability, we only consider a strongly correlated shadowing case in this section. We assume the correlated log-normal components have identical statistics and the correlation coefficient between any two components equals one. Let $\mathcal{E}$ denote this strong correlated case. Under such condition (correlation coefficient = 1), the joint pdf of $p_{o1}, \ldots, p_{on}$ is then reduced to

$$f_{p_{o1} \cdots p_{on}}(p_{o1}, \ldots, p_{on}; \mathcal{E}) = f_{p_{on}}(p_{on}) \prod_{i=1, i\neq j}^{n} \delta(p_{oi} - p_{oj}) \tag{29}$$

where $\delta(x - y)$ is defined to be one for $x = y$, and zero otherwise [15]. Supposing one of the shadowing components is given, such as p_{on}, then the characteristic function of the pdf of the joint instantaneous interference power can be written as

$$\Psi_{p_n}(s \mid p_{o1}, \ldots, p_{on}; \mathcal{E}) = \left(\frac{1}{1 + s p_{on}} \right)^n. \tag{30}$$

The conditional pdf of the joint interference power is then obtained by performing the inverse Laplace transform of (30).

$$\begin{aligned} f_{p_n}(p_n \mid p_{o1}, \ldots, p_{on}; \mathcal{E}) &= \mathcal{L}^{-1}\{\Psi_{p_n}(s \mid p_{o1}, \ldots, p_{on}; \mathcal{E})\} \\ &= \left(\frac{1}{p_{on}} \right)^n \frac{p_n^{n-1}}{\Gamma(n)} \exp\left[\frac{-p_n}{p_{on}} \right]. \end{aligned} \tag{31}$$

It has been shown in Appendix C that

$$\begin{aligned} \int_{\frac{1}{\lambda_{\text{th}}}}^\infty & f_{p_n}(zw \mid p_{o1}, \ldots, p_{on}; \mathcal{E}) dz \\ &= \exp\left(\frac{-w}{\lambda_{\text{th}} p_{on}} \right) \sum_{k=0}^{n-1} \frac{w^{k-1}}{k!} \left(\frac{1}{p_{on}\lambda_{\text{th}}} \right)^k. \end{aligned} \tag{32}$$

Substituting (32) into (17), we obtain

$$\begin{aligned} &F(CI \mid n, p_{od}, p_{o1}, \ldots, p_{on}; \mathcal{E}) \\ &= \sum_{k=0}^{n-1} \frac{1}{k!}\left(\frac{1}{p_{on}\lambda_{\text{th}}}\right)^k \int_0^\infty w^k \exp\left(\frac{-w}{\lambda_{\text{th}} p_{on}}\right) f_{p_d}(w \mid p_{od}) dw \\ &= \sum_{k=0}^{n-1} \frac{1}{k!}\left(\frac{1}{p_{on}\lambda_{\text{th}}}\right)^k \int_0^\infty w^k \exp(-\phi w) f_{p_d}(w \mid p_{od}) dw \\ &= \sum_{k=0}^{n-1} \frac{(-1)^k}{k!}\left(\frac{1}{p_{on}\lambda_{\text{th}}}\right)^k \frac{d^k}{d\phi^k}\mathcal{L}\{f_{p_d}(w \mid p_{od}); \phi\} \end{aligned} \quad (33)$$

where $\mathcal{L}\{f_{p_d}(p_d \mid p_{od}; \phi)\}$, the characteristic function of the pdf of the desired signal, is given in (A6). Thus we have

$$\begin{aligned} &F_{\text{exact, cor}}(CI \mid n, p_{od}, p_{o1}, \ldots, p_{on}; \mathcal{E}) \\ &= \sum_{k=0}^{n-1} \frac{(-1)^k}{k!}\left(\frac{1}{p_{on}\lambda_{\text{th}}}\right)^k \\ &\quad \times \frac{d^k}{d\phi^k}\left[\frac{(R_d+1)\exp\left(\frac{-\phi p_{od} R_d}{\phi p_{od}+R_d+1}\right)}{\phi p_{od}+R_d+1}\right]_{\phi=\frac{1}{p_{on}\lambda_{\text{th}}}}. \end{aligned} \quad (34)$$

For the ease of illustration hereafter, let

$$\begin{aligned} &H^{(k)}(p_{on}, p_{od}) \\ &= \frac{d^k}{d\phi^k}\left[\frac{R_d+1}{\phi p_{od}+R_d+1}\exp\left(\frac{-\phi p_{od} R_d}{\phi p_{od}+R_d+1}\right)\right]_{\phi=\frac{1}{p_{on}\lambda_{\text{th}}}}. \end{aligned} \quad (35)$$

We now transform (34) by averaging over the shadowing (2) and (29)

$$\begin{aligned} &F_{\text{exact, cor}}(CI \mid n) \\ &= \int_0^\infty \cdots \int_0^\infty F_{\text{exact, cor}}(CI \mid n, p_{od}, p_{o1}, \ldots, p_{on}; \mathcal{E}) \\ &\quad \times f_{p_{o1}\cdots p_{on}}(p_{o1}, \ldots, p_{on}; \mathcal{E}) dp_{od} dp_{o1} \cdots dp_{on} \\ &= \int_0^\infty \int_0^\infty \sum_{k=0}^{n-1} \frac{1}{k!}\left(\frac{-1}{p_{on}\lambda_{\text{th}}}\right)^k H^{(k)}(p_{on}, p_{od}) \frac{1}{2\pi\sigma_n\sigma_d p_{on} p_{od}} \\ &\quad \times \exp\left[\frac{-(\ln p_{on} - m_n)^2}{2\sigma_n^2}\right] \exp\left[\frac{-(\ln p_{od} - m_d)^2}{2\sigma_d^2}\right] dp_{od} dp_{on}. \end{aligned} \quad (36)$$

Let $p_{od} = \exp(\sqrt{2}y_d\sigma_d + m_d)$ and $p_{on} = \exp(\sqrt{2}y_n\sigma_n + m_n)$. Equation (36) can now be written as a Hermite integration form, which is

$$\begin{aligned} &F_{\text{exact, cor}}(CI \mid n) \\ &= \int_0^\infty \int_0^\infty \sum_{k=0}^{n-1} \frac{1}{k!}\left(\frac{-e^{-(\sqrt{2}y_n\sigma_n + m_n)}}{\lambda_{\text{th}}}\right)^k \\ &\quad \times H^{(k)}(e^{\sqrt{2}y_n\sigma_n + m_n}, e^{\sqrt{2}y_d\sigma_d + m_d}) \\ &\quad \times \frac{1}{\pi}\exp(-y_n^2)\exp(-y_d^2) dy_d dy_n \end{aligned} \quad (37)$$

where $H^{(k)}(,)$ is defined in (35).

IV. Numerical Examples

A. Comparison of Coherent and Incoherent CCI Probability

In this section we assume that the log-normal shadowing components are uncorrelated. We will compare the coherent and incoherent CCI probabilities derived from various approaches described in the previous two sections. The CCI probabilities are computed as a function of the normalized co-channel reuse distance $R_u = D/R$, where R is the cell radius and D is the minimum distance between the cells using the same channel. The following two-slope path loss model:

$$\frac{p_r}{p_t} = \frac{1}{d^a\left(1+\frac{d}{g}\right)^b}, \quad (38)$$

is used to describe the path loss of the area mean power of signals in microcellular systems, where p_t is the transmitted power, p_r is the received power, and d is the distance between transmitter and receiver, g the turning point, a the basic attenuation coefficient, and b is the additional attenuation coefficient. In our discussion, we assume that the transmitted power from the desired source is the same as the one from the interfering sources. Let ϵ_d be the received area mean power of the desired signal and ϵ_n be the composite received power from the n interfering sources. When a receiver is on the cell boundary and away from the interfering base station with a distance of D, the relation among the R_u, ϵ_d, and ϵ_n can be expressed as

$$\frac{\epsilon_d}{\epsilon_n} = K\frac{R_u^a(G+R_u)^b}{(G+1)^b} \quad (39)$$

where $G = g/R$. The same condition is also used in [1]. The reason we use it in this paper is because we can make a fair comparison between the exact and the approximate analysis. Note that if the Schwartz-Yeh's approximation technique is used, $K = 1/\exp\left(\sum_{i=1}^n G_{i1}\right)$, where G_{i1} is a correction factor on the composite interfering area mean defined in [5] and [10]; on the other hand, if the proposed exact analysis is used, $K = 1$.

The Hermite integration form, as mentioned previously, can significantly reduce the time for computing the CCI probability. With Hermite polynomial, the time consuming integration is replaced by summation. Combining (27) and (39), we obtain the following exact expression of incoherent CCI probability conditional on n, the number of interferers:

$$\begin{aligned} &F_{\text{exact, ind}}(CI \mid n) \\ &= \int_0^\infty \cdots \int_0^\infty \frac{G'(y_d, y_1, \ldots, y_n)}{\sqrt{\pi}^{n+1}} \\ &\quad \times \exp[-(y_d^2 + y_1^2 + \cdots + y_n^2)] dy_d dy_1 \cdots dy_n \\ &= \sum_{k_n=1}^{h_n} \cdots \sum_{k_0=1}^{h_0} \frac{G'(x_{k_0}, x_{k_1}, \ldots, x_{k_n})}{\sqrt{\pi}^{n+1}} w_{k_0} \cdots w_{k_n} + R_m \end{aligned} \quad (40)$$

where the weight factor w_{k_i} at the sample point x_{k_i} of the h_ith-order Hermite polynomial can be found in [11], and R_m is a very small value; $G'()$ is obtained from (28) by replacing

$$f_{i\mathrm{coh}}(u_2) = 1 - Q_1\left(\sqrt{2R_d}, \cdot\sqrt{\left(K\frac{R_u^a(G+R_u)^b}{(G+1)^b}\right)^{-1} 2(R_d+1)\lambda_{\mathrm{th}} \exp\left[-\sqrt{2(\sigma_d^2+\sigma_n^2)}u_2\right]}\right). \quad (43)$$

the area mean power with (38) and (39), as shown in (41) at the bottom of the page, in which D_i is the distance between the receiver and the interfering base station i.

Likewise, combining (39) with (13) and (15), we obtain the approximate incoherent conditional CCI probability

$$F_{i\mathrm{coh}}(CI \mid n) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} f_{i\mathrm{coh}}(u_2)\exp(-u_2^2)du_2 = \frac{1}{\sqrt{\pi}}\sum_{k_0=1}^{h_0} f_{i\mathrm{coh}}(x_{k_0})w_{k_0} + R_m \quad (42)$$

where (43), shown at the top of the page. For the approximate coherent conditional CCI probability, we have

$$F_{\mathrm{coh}}(CI \mid n) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} f_{\mathrm{coh}}(u_2)\exp(-u_2^2)du_2 = \frac{1}{\sqrt{\pi}}\sum_{k_0=1}^{h_0} f_{\mathrm{coh}}(x_{k_0})w_{k_0} + R_m \quad (44)$$

where

$$f_{\mathrm{coh}}(u_2) = \frac{\exp\left(\frac{-R_d H(u_2)}{1+H(u_2)}\right)}{1+H(u_2)} \quad (45)$$

and

$$H(u_2) = \frac{1}{(R_d+1)\lambda_{\mathrm{th}}}\left(K\frac{R_u^a(G+R_u)^b}{(G+1)^b}\right) \times \exp\left(-\sqrt{2(\sigma_d^2+\sigma_n^2)}u_2\right). \quad (46)$$

Here the definitions of x_{k_0}, w_{k_0}, and R_m in (44) and (42) are the same as in (40).

Figs. 1 and 2 compare $F(CI \mid n)$ under various conditions: exact incoherent analysis [(40)], incoherent approximation [(42)], and coherent approximation [(44)]. In these figures, the shadow spreads of the interference are 6 and 12 dB, and the number of interfering sources is assumed to be five ($n = 5$). The results are also tabulated in Table I, in which it is shown that the difference between the coherent and incoherent approximation (20~30%) cannot be ignored. It is also shown that the results of the coherent approximation are closer (difference is about 10%) to those of the exact incoherent case than the incoherent approximation.

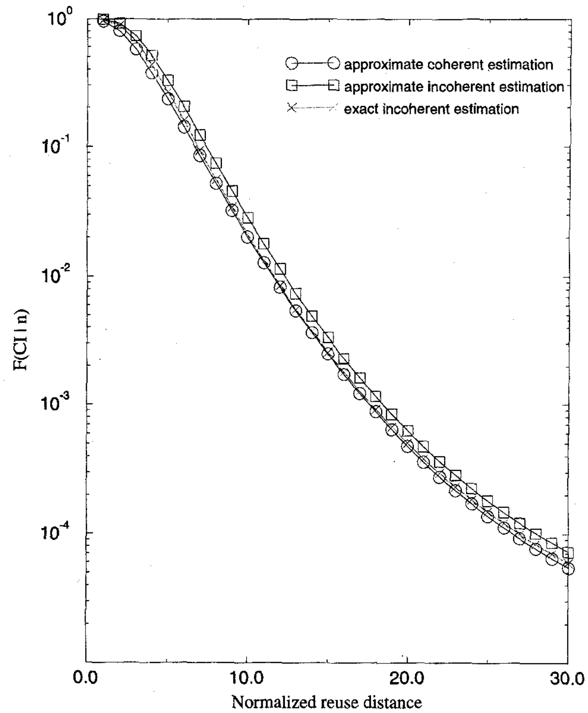

Fig. 1. Comparisons of the coherent and incoherent the conditional CCI probability $F(CI \mid n = 5)$ on a shadowed-Rician/shadowed-Rayleigh channel, where $\sigma_i = \sigma_d = 6$ dB, $\lambda_{\mathrm{th}} = 10$ dB, $R_d = 7$ dB, $a = b = 2$, $g = 0.67R$.

B. Effects of Correlation of Log-Normal Shadowing on the CCI Probability

In Section IV-A, the log-normal shadowing components of the interferers are assumed to be independent. We now

$$G'(y_d, y_1, \cdots, y_n) = \sum_{i=1}^{n} \frac{1}{\prod_{j=1, j\neq i}^{n}\left(1 - \frac{D_i^a\left(1+\frac{D_i}{g}\right)^b}{D_j^a\left(1+\frac{D_j}{g}\right)^b}\exp[\sqrt{2}(y_j\sigma_j - y_i\sigma_i)]\right)} \times \frac{R_d+1}{R_d+1+\frac{1}{\lambda_{\mathrm{th}}}\left(K\frac{R_u^a(G+R_u)^b}{(G+1)^b}\right)\exp[\sqrt{2}(y_d\sigma_d - y_i\sigma_i)]} \times \exp\left[\frac{-\frac{R_d}{\lambda_{\mathrm{th}}}\left(K\frac{R_u^a(G+R_u)^b}{(G+1)^b}\right)\exp[\sqrt{2}(y_d\sigma_d - y_i\sigma_i)]}{R_d+1+\frac{1}{\lambda_{\mathrm{th}}}\left(K\frac{R_u^a(G+R_u)^b}{(G+1)^b}\right)\exp[\sqrt{2}(y_d\sigma_d - y_i\sigma_i)]}\right] \quad (41)$$

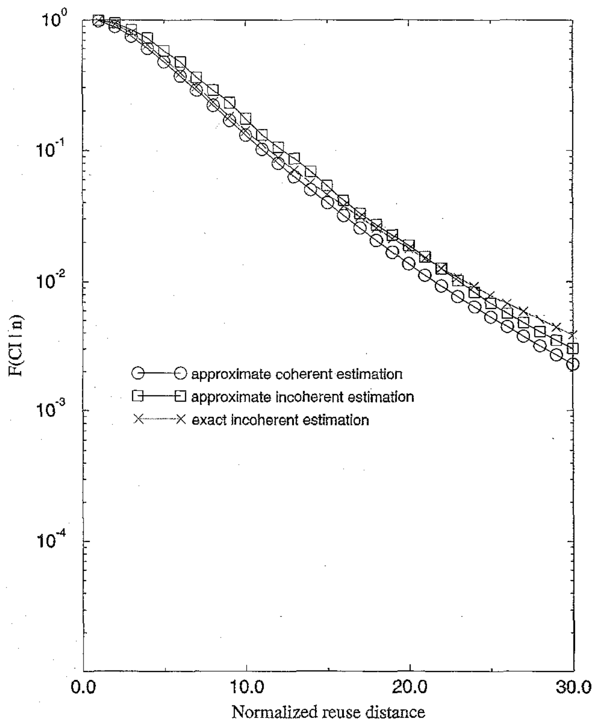

Fig. 2. Comparisons of approximate approach and exact analysis of the conditional CCI probability $F(CI \mid n)$, where $\sigma_i = 12$ dB, $\sigma_d = 12$ dB, $\lambda_{th} = 10$ dB, $R_d = 10$ dB, $a = b = 2$, $g = 0.67R$.

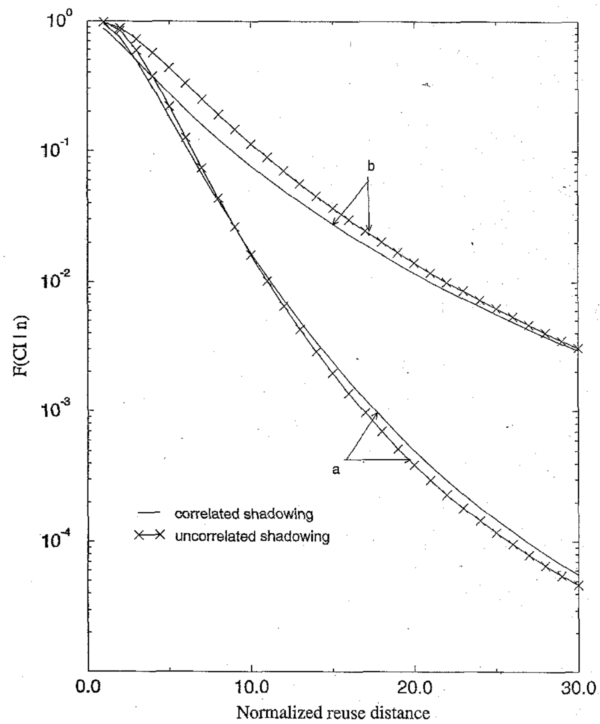

Fig. 3. The effects of correlated log-normal shadowing on conditional CCI probability $F(CI \mid n)$ on a shadowed Rician/shadowed Rayleigh channel with $\sigma_i = 6$ dB (curves a) or 12 dB (curves b), $\sigma_d = 6$ dB, $\lambda_{th} = 10$ dB, $R_d = 7$ dB, $a = b = 2$, $g = 0.67R$.

TABLE I
THE APPROXIMATION AND EXACT ANALYSIS OF THE CONDITIONAL CO-CHANNEL INTERFERENCE PROBABILITY FOR COHERENT AND INCOHERENT MULTIPLE INTERFERING SIGNALS WITH $\sigma_d = 6$ DB, RICIAN FACTOR $R_d = 7$ DB, AND $g = 0.67R$

	$\sigma_i = 6dB$ $F(CI \mid n = 5)$			$\sigma_i = 12dB$ $F(CI \mid n = 5)$		
	approximation		incoherent	approximation		incoherent
R_u	incoherent	coherent	exact analysis	incoherent	coherent	exact analysis
3	0.7360	0.5760	0.6625	0.8414	0.7470	0.7950
4	0.5126	0.3748	0.4329	0.7240	0.6018	0.6421
5	0.3264	0.2326	0.2645	0.5748	0.4727	0.5033
6	0.2022	0.1416	0.1580	0.4732	0.3669	0.3894
7	0.1230	0.0859	0.0939	0.3566	0.2833	0.3002
8	0.0753	0.0523	0.0562	0.2843	0.2184	0.2319
9	0.0455	0.0322	0.0341	0.2289	0.1688	0.1808
10	0.0284	0.0202	0.0211	0.1734	0.1311	0.1408

consider the effect if the shadowing components are correlated. For simplicity, we consider the case $\mathcal{E}$ as described in Section III-B where the shadowing components of the interferers are strongly correlated. Figs. 3 and 4 show both results of the strongly correlated and the independent case. For an environment with pure shadowing only, it has been reported that the correlation between interferers increases the CCI probability (i.e., degrades the performance) and the correlation between the desired signal and interfering signal can decrease the CCI probability [12]. However, in a shadowed-Rician/shadowed-Rayleigh environment, Fig. 3 shows that correlation between interferers increases the CCI probability only if the spread of shadowing is small and the value of the normalized reuse distance is high. If the spread in shadowing is large, this correlation can even yield a better performance. It is also found that in a shadowed-Rayleigh (desired)/shadowed-Rayleigh (interfering) channel (see Fig. 4), regardless of degree of spread in shadowing, this correlation seems always to improve the performance. Similar conclusion was also reached in [13], based on a coherent approximate analysis for a shadowed Rayleigh channel. To double verify our results, we also use the exact analysis to compute the case of shadowed-Rayleigh (desired)/shadowed-Rayleigh (interfering) and compare with Linnartz's exact analysis for that case [4]. The results are shown in Fig. 4. The two identically match. Moreover, the performance impact of the correlation is more significant when the spread of shadowing is large. For example, the performance difference on the CCI probability between the correlated and uncorrelated case (shown in Table II) is within the range 26~37% for 12-dB spread of shadowing and 1~18% for 6-dB spread of shadowing.

C. Comparison of Spectrum Efficiency

The reason we derive the CCI probability in the first place is to compute spectrum efficiency of a cellular system. In the following we give one example to illustrate the relationship between the two. Spectrum efficiency E_s in a cellular mobile

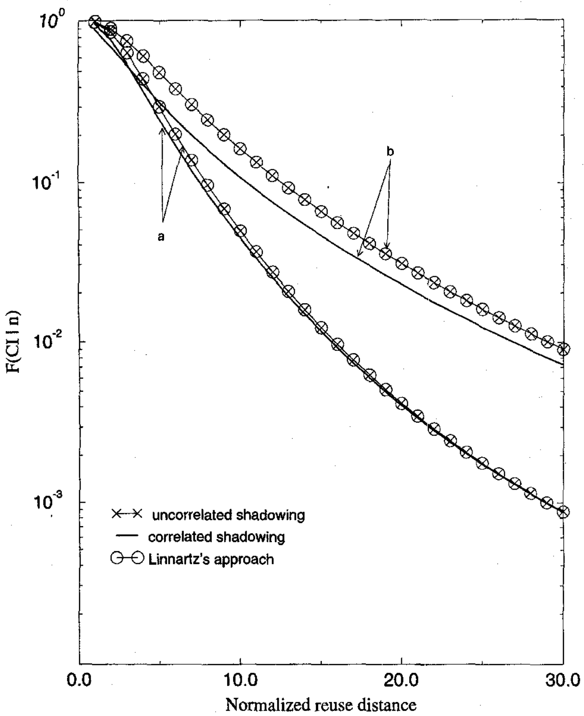

Fig. 4. The effects of correlated log-normal shadowing on conditional CCI probability $F(CI \mid n = 4)$ on a shadowed Rayleigh/shadowed Rayleigh channel with $\sigma_i = 6$ dB (curves a) or 12 dB (curves b), $\sigma_d = 6$ dB, $\lambda_{\mathrm{th}} = 10$ dB, $a = b = 2$, $g = 0.67R$.

TABLE II
EXACT ANALYSIS OF THE CONDITIONAL INCOHERENT CCI PROBABILITY WITH CORRELATED SHADOWING AND UNCORRELATED SHADOWING ON A SHADOWED RICIAN/SHADOWED RAYLEIGH CHANNEL, WHERE $\sigma_d = 6$ DB, $\sigma_i = 6$ DB OR 12 DB, RICIAN FACTOR $R_d = 7$ DB, AND $g = 0.67R$, AND CORRELATION COEFFICIENT EQUALS ONE FOR THE CORRELATED CASE

	$\sigma_i = 6dB$ $F(CI \mid n=4)$		$\sigma_i = 12dB$ $F(CI \mid n=4)$	
R_u	correlated	uncorrelated	correlated	uncorrelated
4	0.3029	0.3706	0.3664	0.5672
5	0.1825	0.2189	0.2737	0.4326
6	0.1100	0.1273	0.2071	0.3280
7	0.0670	0.0742	0.1588	0.2491
8	0.0415	0.0438	0.1232	0.1902
9	0.0261	0.0264	0.0968	0.1464
10	0.0168	0.0162	0.0769	0.1137
11	0.0110	0.0102	0.0616	0.0891
12	0.0073	0.0065	0.0499	0.0705
13	0.0049	0.0043	0.0407	0.0562
14	0.0034	0.0029	0.0334	0.0452

radio system is defined as

$$E_s = \frac{\text{Carried traffic per cell}}{\text{(Total bandwidth) (Cell area)}} = \frac{A_c}{n_c C f_B S_c} (\text{erlang/MHz/km}^2) \qquad (47)$$

where A_c is the carried traffic per cell, n_c is the number of channels in a cell, C is the cluster size of cells sharing the whole allocated frequency spectrum, f_B is the channel bandwidth, and S_c is the covered area of a cell. Note that the carried traffic A_c can be obtained by the offered traffic A and the blocking probability B, i.e. $A_c = A(1 - B)$, where A and B are related by the Erlang-B formula. For a hexagonal system, the cluster size is determined by

$$C = i^2 + j^2 + ij \qquad (48)$$

where i and j are nonnegative integers; the reuse distance R_u is

$$R_u = \sqrt{3C}. \qquad (49)$$

Let the channel utilization be defined by

$$a_c = \frac{A_c}{n_c}. \qquad (50)$$

Then the total co-channel interference probability is given by

$$F(CI) = \sum_{i=1}^{n} F(CI \mid i) \binom{n}{i} a_c^i (1 - a_c)^{n-i} \qquad (51)$$

where $F(CI \mid i)$ is the CCI probability conditional on i, the number of active interferers, n is the total number of interferers, and $\binom{n}{i} a_c^i (1 - a_c)^{n-i}$ is equal to the probability of i independent and identical interferers being active.

Supposing $C = 7$, $f_B = 10$ kHz, and $S_a = 1$ km^2, the comparisons of the coherent approximation, incoherent approximation, and exact analysis of the total co-channel interference probability versus channel utilization are shown in Fig. 5. At the point of 10% of the CCI probability, the channel utilizations are 0.287, 0.208, and 0.275 for coherent approximation, incoherent approximation, and exact analysis. In terms of spectrum efficiency, they are 4.10, 2.97, and 3.93 (erlangs/MHz/km^2), implying that the coherent approximation is an optimistic analysis, and incoherent approximation a pessimistic analysis.

V. CONCLUDING REMARKS

In a diverse environment of a microcellular system, an incoherent addition for summing interfering signals is a more realistic assumption. We have analyzed the incoherent CCI probability and compared the results with the coherent case on a microcellular system with a shadowed-Rician/shadowed-Rayleigh channel. Three approaches for calculating the CCI probability are discussed in this paper: coherent approximation, incoherent approximation, and exact analysis. The results indicate that the approximate incoherent analysis produces an error of 20~30%. It is surprising to find that the coherent approximation produces a closer result to the exact incoherent analysis (10% difference).

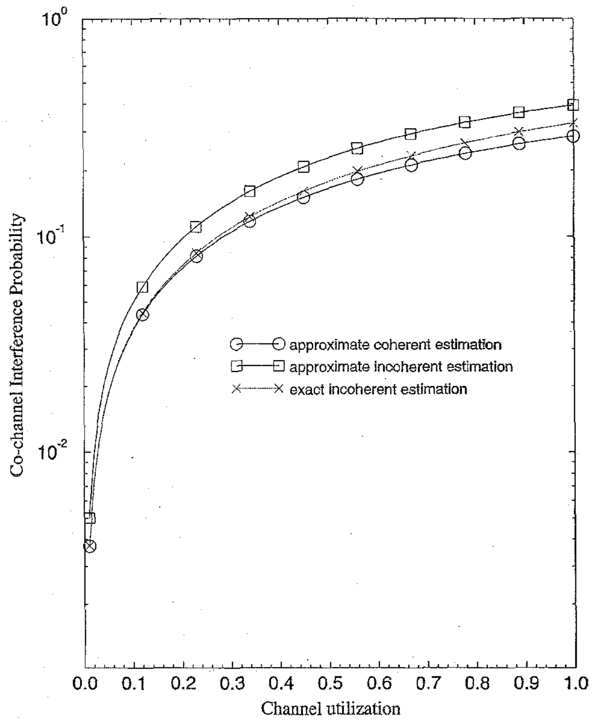

Fig. 5. Comparison of the CCI probability versus channel utilization for using coherent approximation, incoherent approximation and exact analysis with cluster size $C = 7$, where there are five identical shadowed Rayleigh interferers and a shadowed Rician desired signal with $\sigma_i = 6$ dB, $\sigma_d = 6$ dB, $\lambda_{\text{th}} = 10$ dB, $a = b = 2$, $g = 0.67R$.

The effect of correlation between the shadowing components of the interferers is also studied. The case we studied is a strongly correlated case with correlation coefficient which is equal to one. It is found that in a shadowed-Rician/shadowed-Rayleigh channel, this kind of correlation increases the CCI probability for cases having a small spread in shadowing and large reuse distance. However, if the spread in shadowing is large, this correlation can even improve the performance. It is also found that in a shadowed-Rayleigh (desired)/shadowed-Rayleigh (interfering) channel, regardless of degree of spread in shadowing, this correlation seems always to improve the performance. We can easily combine the techniques to study the cases where some interferers are independent while the others are strongly correlated.

As for the effects of the correlation between the shadowing desired signals and the shadowing interfering signals, it has been shown that the increased correlation improves the performance of the CCI probability on a shadowed Rayleigh channel [13] or a pure log-normal channel [12]. How this kind of correlation will affect the performance in a microcellular environment is a topic left for future research.

Appendix A
Characteristic Function of the PDF of the Desired Signal Power in a Rician Fading Environment

Recall in Rician fading environment, the pdf of the desired signal power is noncentral Chi square with two degree of freedom, as shown in (1). The characteristic function of the pdf of the desired signal power is simply the Laplace transformation of the (1) in Section II; that is

$$\begin{aligned}\mathcal{L}\{f_{p_d}(p_d \mid p_{od}); \phi\} &= \int_0^\infty \exp(-\phi p_d)\frac{R_d+1}{p_{od}}\exp\left[\frac{-(R_d+1)(s^2+2p_d)}{2p_{od}}\right] \\ &\quad\times I_0\left(\frac{(R_d+1)s\sqrt{2p_d}}{p_{od}}\right)dp_d \\ &= \frac{R_d+1}{p_{od}}\exp\left[\frac{-(R_d+1)s^2}{2p_{od}}\right] \\ &\quad\times\int_0^\infty \exp\left[\frac{-(\phi p_{od}+R_d+1)p_d}{p_{od}}\right] \\ &\quad\times I_0\left(\frac{(R_d+1)s\sqrt{2p_d}}{p_{od}}\right)dp_d. \end{aligned} \tag{A1}$$

Let

$$\eta = \frac{(R_d+1)s}{\phi p_{od}+R_d+1} \tag{A2}$$

and

$$\Lambda = \phi p_{od} + R_d. \tag{A3}$$

Then (A1) becomes

$$\begin{aligned}\mathcal{L}\{f_{p_d}(p_d \mid p_{od}); \phi\} &= \frac{R_d+1}{p_{od}}\exp\left[\frac{-(R_d+1)s^2}{2p_{od}}\right] \\ &\quad\times\int_0^\infty \exp\left[\frac{-(\Lambda+1)p_d}{p_{od}}\right]I_0\left(\frac{(\Lambda+1)\eta\sqrt{2p_d}}{p_{od}}\right)dp_d \\ &= \frac{R_d+1}{\Lambda+1}\exp\left[\frac{-(R_d+1)s^2+(\Lambda+1)\eta^2}{2p_{od}}\right]\int_0^\infty \frac{\Lambda+1}{p_{od}} \\ &\quad\times\exp\left[\frac{-(\Lambda+1)(\eta^2+2p_d)}{2p_{od}}\right]I_0\left(\frac{(\Lambda+1)\eta\sqrt{2p_d}}{p_{od}}\right)dp_d. \end{aligned} \tag{A4}$$

By substituting (A2) and (A3) into (A4), and using the definition of Rician factor [1, (1)]

$$R_d = \frac{s^2(R_d+1)}{2p_{od}} \tag{A5}$$

then the characteristic function of the desired signal power in a Rician fading channel can be expressed by

$$\begin{aligned}\mathcal{L}\{f_{p_d}(p_d \mid p_{od}); \phi\} &= \frac{R_d+1}{\phi p_{od}+R_d+1}\exp\left[\frac{-s^2(R_d+1)\phi}{2(\phi p_{od}+R_d+1)}\right] \\ &= \frac{R_d+1}{\phi p_{od}+R_d+1}\exp\left[\frac{-\phi p_{od}R_d}{\phi p_{od}+R_d+1}\right]. \end{aligned} \tag{A6}$$

Appendix B
Derivation of (17)

By letting $z = p_n/p_d$ and $w = p_d$, the joint pdf of z and w can be expressed in terms of the one of p_n and p_d as

$$f_{zw}(z,w) = f_{p_n p_d}(zw, w)w. \tag{B1}$$

Assume the pdfs of the desired signal power and interference power are independent. We can represent the marginal pdf of z as

$$f_z(z) = \int_0^\infty f_{p_n p_d}(zw, w) w dw = \int_0^\infty f_{p_n}(zw) f_{p_d}(w) w dw. \tag{B2}$$

Thus, (17) can be derived as follows:

$$\begin{aligned} \mathrm{Prob}\left[\frac{p_d}{p_n} < \lambda_{\mathrm{th}}\right] &= \mathrm{Prob}\left[\frac{p_n}{p_d} \geq \frac{1}{\lambda_{\mathrm{th}}}\right] \\ &= \int_{\frac{1}{\lambda_{\mathrm{th}}}}^\infty f_z(z) dz \\ &= \int_{\frac{1}{\lambda_{\mathrm{th}}}}^\infty \left[\int_0^\infty f_{p_n}(zw) f_{p_d}(w) w dw\right] dz \\ &= \int_0^\infty \left[\int_{\frac{1}{\lambda_{\mathrm{th}}}}^\infty f_{p_n}(zw) dz\right] f_{p_d}(w) w dw. \end{aligned} \tag{B3}$$

APPENDIX C
DERIVATION OF (32)

Recall [14, (3.351)], which is

$$\int_\alpha^\infty x^n \exp(-\beta x) dx = \exp(-\alpha\beta) \sum_{k=0}^n \left(\frac{n!}{k!}\right) \frac{\alpha^k}{\beta^{n-k+1}}. \tag{C1}$$

Combining (31) and (C1), we have

$$\begin{aligned} &\int_{\frac{1}{\lambda_{\mathrm{th}}}}^\infty f_{p_n}(zw \mid p_{od}, p_{o1}, \ldots, p_{on}; \mathcal{E}) dz \\ &= \int_{\frac{1}{\lambda_{\mathrm{th}}}}^\infty \left(\frac{1}{p_{on}}\right)^n \frac{(zw)^{n-1}}{\Gamma(n)} \exp\left[\frac{-wz}{p_{on}}\right] dz \\ &= \left(\frac{1}{p_{on}}\right)^n \frac{w^{n-1}}{(n-1)!} \int_{\frac{1}{\lambda_{\mathrm{th}}}}^\infty z^{n-1} \exp\left(\frac{-wz}{p_{on}}\right) dz \\ &= \left(\frac{1}{p_{on}}\right)^n \frac{w^{n-1}}{(n-1)!} \exp\left(\frac{-w}{\lambda_{\mathrm{th}} p_{on}}\right) \\ &\quad \times \sum_{k=0}^{n-1} \frac{(n-1)!}{k!} \frac{\lambda_{\mathrm{th}}^{-k}}{(w/p_{on})^{n-k}} \\ &= \exp\left(\frac{-w}{\lambda_{\mathrm{th}} p_{on}}\right) \sum_{k=0}^{n-1} \frac{w^{k-1}}{k!} \left(\frac{1}{p_{on}\lambda_{\mathrm{th}}}\right)^k. \end{aligned} \tag{C2}$$

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