Co-Channel Interference Analysis of Shadowed Rician Channels

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***Abstract*—This paper presents an exact analysis of co-channel interference (CCI) in a shadowed-Nakagami (desired)/shadowed-Rician (interfering) channel. Because a Rician distribution can be closely approximated by a Nakagami distribution [1], the proposed analysis can be also viewed as a close approximation of a shadowed-Rician (desired)/shadowed-Rician (interfering) model. The model provided in this paper includes many flexibilities—different Rice factors, different shadowing spreads, and different transmitted powers for desired and interfering signals. Therefore, it is a powerful tool for determining cell sizes and power levels in mixed micro/macro cellular systems.**

***Index Terms*—Cellular radio system, co-channel interference, Nakagami fading, Rician fading, shadowing.**

I. Introduction

CO-CHANNEL interference (CCI) probability, defined as the probability of an instantaneous signal-to-interference (S/I) ratio below the required threshold at the receiver, is affected by the statistical propagation model of the desired signal and interference. Ideally, we would like to use a model which describes both the desired and interfering signals as a shadowed Rician distribution. The problem, however, is that the exact analysis of such a shadowed-Rician (desired)/shadowed-Rician (interfering) model containing different Rice factors, different shadowing spreads, and different transmitted powers has not been found.

Several works on the *exact analysis* of CCI probability in a shadowed Rician channel have been proposed [2]–[6]. All have fallen short from the ideal situation described above. In this letter, we present an exact analysis of a shadowed-Nakagami (desired)/shadowed-Rician (interfering) channel. Because a Rician distribution can be closely approximated by a Nakagami distribution [1], the proposed analysis can also be viewed as a close approximation of the ideal shadowed-Rician (desired)/shadowed-Rician (interfering) model. The model provided in this paper includes many flexibilities—different Rice factors, different shadowing spreads, and different transmitted powers, thereby allowing us to study more complicated micro/macro cellular systems.

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II. Microcellular Channel Model

The radio propagation environment of a cellular channel is usually described by a three-stage channel model. The key parameters of the model are: 1) path loss, 2) shadowing, and 3) amplitude fading. In urban microcells, a two-slope path loss model is usually used, in which the received area mean power Υ_d is given by

$$\Upsilon_d = \frac{P_t C}{r_d^{\alpha}(1 + r_d/g)^{\beta}} \tag{1}$$

where P_t is the transmitted power; C is the constant including the effect of antenna gain; r_d is the distance between the transmitter and the receiver; g is the turning point; α is the basic attenuation coefficient; and β is the additional attenuation coefficient.

To characterize the shadowing effect, a log–normal distribution is used to describe the slowly-varying local mean p_{od}. That is,

$$f_{p_{od}}(p_{od}) = \frac{1}{\sqrt{2\pi}\sigma_d p_{od}} \exp\left[\frac{-(\ln p_{od} - \ln \Upsilon_d)^2}{2\sigma_d^2}\right] \tag{2}$$

where σ_d is the shadowing spread and Υ_d is the area mean power determined by (1).

There are two models for amplitude fading—Rician fading and Nakagami fading. Given local mean p_{od}, the conditional pdf of the signal power y_d in Rician-fading channel is described by a noncentral Chi-square distribution with 2 degrees of freedom (d.o.f.); that is,

$$f_{y_d}(y_d|p_{od}) = \frac{K_d + 1}{p_{od}} \exp\left[-K_d - \frac{(K_d+1)y_d}{p_{od}}\right] \cdot I_0\left[\sqrt{\frac{4K_d(K_d+1)y_d}{p_{od}}}\right] \tag{3}$$

where K_d is the Rice factor.

In Nakagami fading model, the pdf of the instantaneous signal power y_d is

$$f_{y_d}(y_d|p_{od}) = \left(\frac{m_d}{p_{od}}\right)^{m_d} \frac{{y_d}^{m_d - 1}}{\Gamma(m_d)} \exp\left[-\frac{m_d y_d}{p_{od}}\right] \tag{4}$$

where $\Gamma(.)$ is the gamma function and the constant m_d is called the Nakagami parameter.

The method of approximating a Rician random variable by a Nakagami random variable can be justified by the equivalence of the mean and the variance through the following relations [1]:

$$m_d = \frac{(K_d+1)^2}{2K_d + 1}. \tag{5}$$

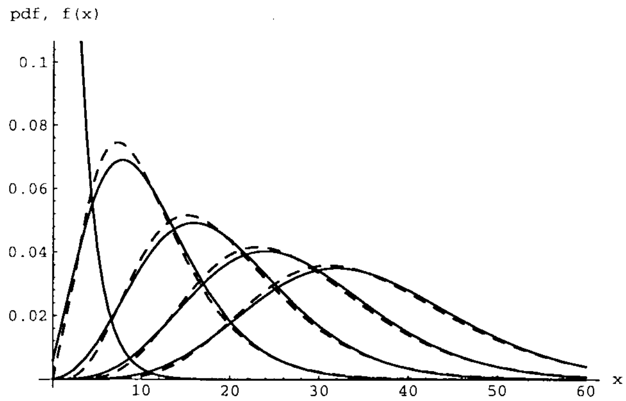

Fig. 1. Comparison of Rician fading model (noncentral Chi-square distribution with 2 d.f., solid lines) and Nakagami fading model (Gamma distribution, dashed lines), where $m_d = 1, 3, 5, 7, 9$; the scattering power is equal to two.

Fig. 1 compares a Rician model with the approximation achieved by using (5) in a Nakagami model. For the case $m_d = 1$, Nakagami model and Rician model become identical; for other m_d, we see the larger the value of m_d, the better the approximation.

III. CCI Probability on a Shadowed-Nakagami/Shadowed-Rician Channel

CCI probability $P(CI)$ is defined as

$$P(CI) \triangleq \text{Prob}[y_d/y_I < \lambda_{\text{th}}] \tag{6}$$

where y_d is the desired signal power, y_I the total interference power, and λ_{th} the required threshold at the receiver. In this paper, y_I is assumed to be the accumulated power of uncorrelated co-channel interferers. Considering (4), we calculate the conditional CCI probability in terms of the local mean power of the desired signal p_{od} and the interfering signals $p_I = \{p_{o1}, \cdots, p_{on}\}$ as

$$P(CI|p_{od}, p_I) = 1 - \int_0^\infty \left[\int_{\lambda_{th}}^\infty \left(\frac{m_d}{p_{od}}\right)^{m_d} \frac{y_d {y_I}^{m_d-1}}{\Gamma(m_d)} \cdot \exp\left(-\frac{m_d y_d y_I}{p_{od}}\right) dy_d\right] f_{y_I}(y_I|p_I) y_I \, dy_I \tag{7}$$

where $f_{y_I}()$ is the pdf of y_I. Recalling [7, eq. 3.351], we can simplify the internal integration over y_d in (7) as

$$\int_{\lambda_{\text{th}}}^\infty \left(\frac{m_d}{p_{od}}\right)^{m_d} \frac{y_d {y_I}^{m_d-1}}{\Gamma(m_d)} \exp\left[-\frac{m_d y_d y_I}{p_{od}}\right] dy_d = \exp\left(\frac{-y_I \lambda_{\text{th}}}{p_I}\right) \sum_{k=0}^{n-1} \frac{{y_I}^{k-1}}{k!} \left(\frac{\lambda_{\text{th}}}{p_I}\right)^k. \tag{8}$$

Let

$$\phi = \frac{\lambda_{\text{th}} m_d}{p_{od}} \tag{9}$$

and combine (7) with (8). We then have

$$P(CI|p_{od}, p_I) = 1 - \sum_{k=0}^{m_d-1} \frac{(-\phi)^k}{k!} \frac{d}{d\phi^k} \int_0^\infty \exp(-\phi y_I) f_{y_I}(y_I|p_I) \, dy_I. \tag{10}$$

Note that the internal integration over y_I is equivalent to find the characteristic function of random variable y_I given p_I. It can be shown that the characteristic function of the signal power y_i in a Rician-fading channel is given by [3]

$$\int_0^\infty \exp(-\phi y_i) f_{y_i}(y_i|p_{oi}) \, dy_i = \frac{K_i + 1}{K_i + 1 + \phi p_{oi}} \exp\left[-\frac{K_i \phi \Omega_i}{K_i + 1 + \phi p_{oi}}\right]. \tag{11}$$

Since y_I is the sum of the n independent random variables y_i, the characteristic function of y_I is given by

$$\int_0^\infty \exp(-\phi y_I) f_{y_I}(y_I|p_I) \, dy_I = \prod_{i=1}^n \frac{K_i + 1}{K_i + 1 + \phi p_{oi}} \exp\left[-\frac{K_i \phi p_{oi}}{K_i + 1 + \phi p_{oi}}\right]. \tag{12}$$

Substituting (12) into (10), we obtain

$$P(CI|p_{od}, p_I) = 1 - \sum_{k=0}^{m_d-1} \frac{(-\phi)^k}{k!} \cdot \left[\frac{d}{d\phi^k} \prod_{i=1}^n \frac{K_i + 1}{K_i + 1 + \phi p_{oi}} \exp\left(-\frac{\phi p_{oi}}{K_i + 1 + \phi p_{oi}}\right)\right]. \tag{13}$$

The derivative in (13) can be calculated by mathematical packages, such as Mathematica or Maple, so that the closed form can be obtained. We transform (13) by averaging over local mean power, $\Omega_d, \Omega_1, \cdots, \Omega_n$, any two of which are assumed to be mutually independent. Then

$$P(CI) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty P(CI|p_{od}, p_{o1}, \cdots, p_{on}) \cdot \left[\prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma_i p_{oi}} \exp\left[\frac{-(\ln p_{oi} - \ln \Upsilon_i)^2}{2\sigma_i^2}\right]\right] \cdot \frac{1}{\sqrt{2\pi}\sigma_d p_{od}} \exp\left[\frac{-(\ln p_{od} - \ln \Upsilon_d)^2}{2\sigma_d^2}\right] dp_{od} \cdots dp_{on}. \tag{14}$$

Note that (14) can be calculated by using the Hermite polynomial approach, which requires only summation and no integration [8].

In summary, the effects of Rice factors of desired and interfering signals are captured by K_i and m_d in (13), respectively. The effects of shadowing spreads and the dual-slope path loss model are included in (14). Hence, the proposed approach allows us to analyze a more complicated scenario, where all signals may have different Rice factors, different shadowing spreads, and different transmit powers.

IV. Numerical Results

Consider a one-dimensional cellular structure with some existing cells. Now we want to add a larger cell (see Fig. 2). We assume the radius of each equal-sized micro-cell is R and the one of macrocell A' is R' (assuming reuse factor $N = 3$ in our case, where cells A and A' are assigned with the same

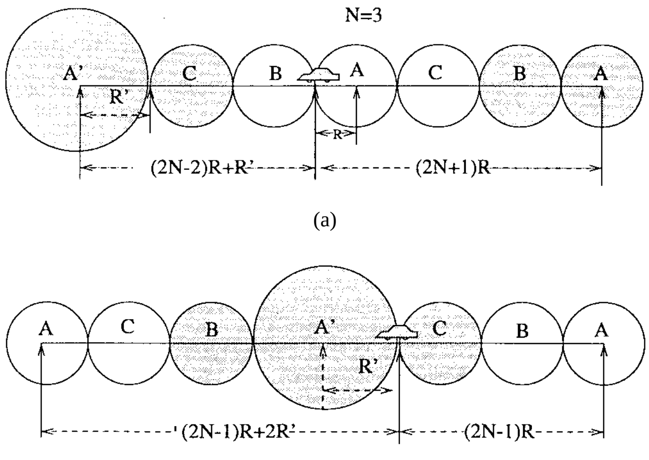

Fig. 2. A one-dimensional mixed-cell architecture, where: (a) indicates the macro-cell imposes the interference on the micro-cells and (b) shows the opposite situation.

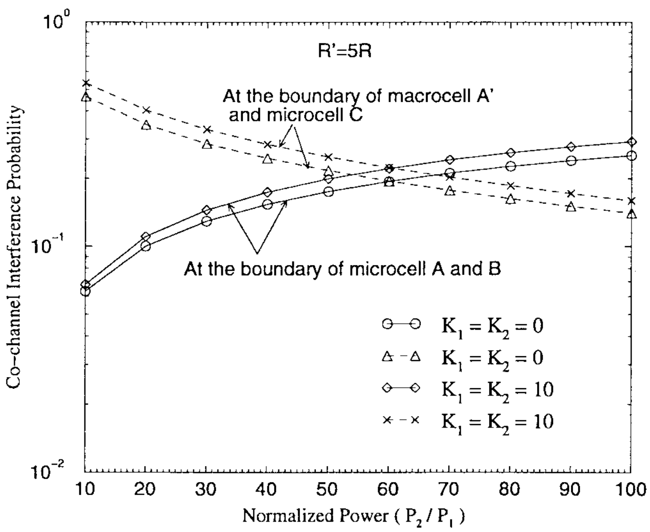

Fig. 3. The CCI probability performance versus the normalized transmit power of the macrocell on a one-dimension mixed cell architecture ($R' = 5R$), where $K_d = 4.45$; $\sigma_d = 4$ dB; two different interferers with $\sigma_i = 4$ dB.

channel set). One major issue is to determine the cell size and the transmit power of the macrocell. Obviously we need to increase the transmit power of the larger cell to cover a larger area. However, this will also induce additional co-channel interference. Thus we would like to know the suitable range of the transmit power of the larger cell so that both: 1) the CCI performance of the small cells and 2) the CCI performance of the larger cell will not be violated.

Suppose the cell radius of macro-cell is $R' = 5R$ and $R' = 3R$. Figs. 3 and 4 show the CCI probability versus normalized power, which is defined as the ratio of the transmit powers of the macrocell P_2 to the one of the microcell P_1 (i.e., P_2/P_1). From Fig. 3, the case $R' = 5R$, it is observed that for 20% CCI probability, there is only one solution to the transmit power (i.e., $P_2/P_1 = 60$). If we set the required performance to 10% CCI probability, there is no solution to the transmit power that can meet the two requirements

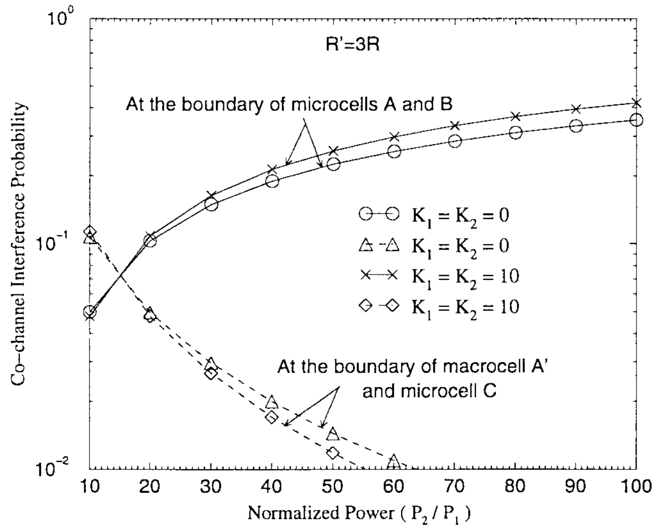

Fig. 4. The CCI probability performance versus the normalized transmit power of the macrocell on a one-dimension mixed cell architecture ($R' = 3R$), where $K_d = 4.45$, $\sigma_d = 4$ dB; two interferers are with different distances and $\sigma_i = 4$ dB.

simultaneously. For another case $R' = 3R$ (see Fig. 4), under the 10% CCI requirement, there are numerous solutions to the transmit power (all $10 < P_2/P_1 < 20$ can satisfy). In summary, we can say the size of the larger cell cannot be too large; otherwise, the two conditions mentioned previously will not be met simultaneously.

V. Concluding Remarks

An exact analytical method of calculating the CCI probability on a shadowed-Nakagami (desired)/shadowed-Rician (interfering) channel is proposed. This model allows the line-of-sight components in both the desired and interfering signals, allows different Rice factors, different shadowing spreads, and different transmitted powers (i.e., irregular cell sizes). The proposed analytical technique allows us to study more complicated situations which may not be tractable with other approaches.

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