Probability of False Power Control Command in CDMA Systems Subject to Measurement Errors

Li-Chun Wang and Chih-Wen Chang

Abstract— **In this letter we introduce a new performance measurement, the probability of false power control command (P_{FC}), to evaluate the impact of measurement errors on the closed-loop power control in code division multiple access systems. We derive a closed-form expression for P_{FC}. As compared to the variable-step-sized power control, we find that the fixed-step-sized CLPC scheme, thanks to its non-linear operation in the up/down control scheme, is less sensitive to the measurement error.**

Index Terms— **CDMA, power control, false power control command.**

I. INTRODUCTION

ACCURATE power control is one of key technologies to achieve high capacity in code division multiple access (CDMA) systems. Power control errors may result from many factors [1], such as loop delay [2], quantization errors [3], multi-path fading [4], [5], link-quality measurement errors [6], and feedback errors [7]. Although the closed-loop power control (CLPC) in the CDMA system has been extensively studied in the literature [3], [6], [8], fewer papers have analyzed the performance of the CLPC scheme subject to measurement errors.

Previous works about the impact of measurement errors on power control can be summarized as follows. In [6], the author investigated the impact of measurement errors on the open-loop power control. In [1], the filtering effect of the measurement scheme is discussed for the CDMA systems. In [8]–[10], the issue of joint minimization of signal-to-interference (SIR) measurement errors and power control errors are investigated as a stochastic control problem, but the SIR measurement errors in [8], [9] are modelled as the white Gausssian noise. However, in [11], it has been found that measurement errors tend to be log-normally distributed in the cellular channel with the Rayleigh fading and shadowing.

The goal of this letter is to develop a simple and accurate analytical model to evaluate the impact of log-normally distributed SIR measurement errors on the CLPC of CDMA systems. We introduce a new performance measurement, the probability of false power control command (P_{FC}), to evaluate the impact of measurement errors on the CLPC. The motivation of introducing P_{FC} is from the fact that the feedback power control command is the key to the accuracy of the CLPC scheme. This letter will present a closed-form expression for P_{FC}.

The rest of this letter is organized as follows. In Section II, we define the probability P_{FC}. Section III derives the closed-form expression for P_{FC} subject to measurement errors. Section IV shows analytical and simulation results. Section V gives concluding remarks.

II. DEFINITION OF PROBABILITY OF FALSE POWER CONTROL COMMAND

Because the accuracy of a power control command strongly influences the performance of the closed-loop power control, it is important to investigate the impact of measurement errors on the power control command itself. Figs. 1(a) and 1(b) illustrates how measurement errors influence the accuracy of power control commands in the CLPC scheme. The size of the gap between the received SIR and the target SIR also plays an important role. In some cases, a received signal with a smaller measurement error may be even more likely to issue a false power control command than that with a larger measure error. Take Fig. 1(a) as an example. Signal A has a smaller measurement error e_a than signal B does, i.e. $e_a < e_b$. Nevertheless, signal A is closer to the target SIR than signal B. Thus, after considering the impact of measurement errors, signal A' (the original signal A plus a small measurement error e_a) exceed the target SIR, but signal B' (the original signal B plus a large measurement error e_b) is still below the target SIR. In this example, it is the signal with a smaller measurement error (i.e. signal A) that causes the CLPC to issue a false power control command. Based on this observation, we are motivated to define a new performance measurement — the probability of false power control command (P_{FC}) — to characterize the effect of measurement errors in the CLPC.

The probability of the false power control command is defined as follows:

$$\begin{aligned} P_{FC} = \text{Prob}\{ \quad & sgn \quad (SIR_{T(dB)} - SIR_{M(dB)}) \\ \neq \quad & sgn \quad (SIR_{T(dB)} - SIR_{o(dB)})\} \ , \end{aligned} \quad (1)$$

where $sgn(x)$ is the operator to choose the sign of x, SIR_M is the measured SIR, SIR_o is the original SIR value without measurement errors, and the SIR_T is the target SIR. According to the definition of (1), if a measurement error embedded in SIR_M makes the sign of $(SIR_{T(dB)} - SIR_{M(dB)})$ different from that of $(SIR_{T(dB)} - SIR_{0(dB)})$, the CLPC scheme will issue a false power control command. Unlike the traditional concept of the absolute measurement errors, the definition of (1) implies a new concept of relative measurement errors,

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Manuscript received April 6, 2004. The associate editor coordinating the review of this letter and approving it for publication was Ezio Biglieri. This work was supported jointly by the Lee and MTI center for networking research, and the National Science Council Taiwan R.O.C under the contracts 89-E-FA06-2-4, EX-91-E-FA06-4-4, 91-2219-E-009-016 and 92-2219-E-009-026.

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Digital Object Identifier 10.1109/LCOMM.2005.04002.

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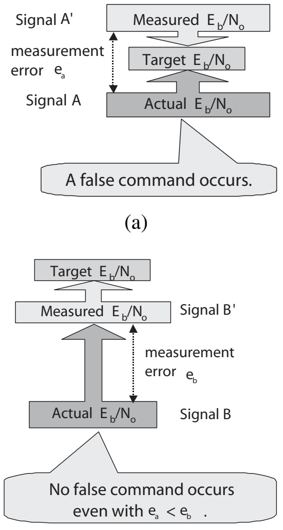

(a)

(b)

Fig. 1. Effect of measurement errors on the closed-loop power control, where the measurement error in (a) e_a is smaller than e_b in (b).

which means that both the amount of measurement errors and the gap size between SIR_T and SIR_M will affect the accuracy of the CLPC scheme.

III. PROBABILITY OF FALSE POWER CONTROL COMMAND

In this section, we derive an analytical formula to calculate the probability of false power control command in terms of measurement errors and power control errors. To begin with, denote ξ_p and ξ_m as power control error and measurement error in the dB domain, respectively. Power control error ξ_p is defined as the difference between the target SIR (denoted as SIR_T) and the ideal received signal (denoted as SIR_o) in the dB domain. That is,

$$\xi_p = SIR_{T(dB)} - SIR_{o(dB)} \ . \tag{2}$$

Note that SIR_o contains no measurement errors. In (2), a negative value of ξ_p implies the necessity to decrease transmission power, and vice versa.

Because the measured SIR (denoted as SIR_M) is the sum of the ideal SIR and the measurement error ξ_m, we have

$$SIR_{M(dB)} = SIR_{o(dB)} + \xi_m \ . \tag{3}$$

Substituting (2) into (3), we can obtain

$$SIR_{M(dB)} = SIR_{T(dB)} + \xi_p + \xi_m \ . \tag{4}$$

Substituting (2) and (4) into (1), we find that a false power control command occurs when the following condition is sustained:

$$\begin{aligned} &\operatorname{sgn}(SIR_{T(dB)} - (SIR_{T(dB)} + \xi_m + \xi_p)) \\ &\quad \neq \operatorname{sgn}(SIR_{T(dB)} - (SIR_{T(dB)} + \xi_p)) \ . \end{aligned} \tag{5}$$

Thus the probability of the false power control command can be written as

$$\begin{aligned} P_{FC} &= \text{Prob}\{\operatorname{sgn}(\xi_m + \xi_p) \neq \operatorname{sgn}(\xi_p)\} \\ &= \text{P}\{\xi_p + \xi_m < 0,\ \xi_p > 0\} + \\ &\quad\ \text{P}\{\xi_p + \xi_m > 0,\ \xi_p \leq 0\} \ . \end{aligned} \tag{6}$$

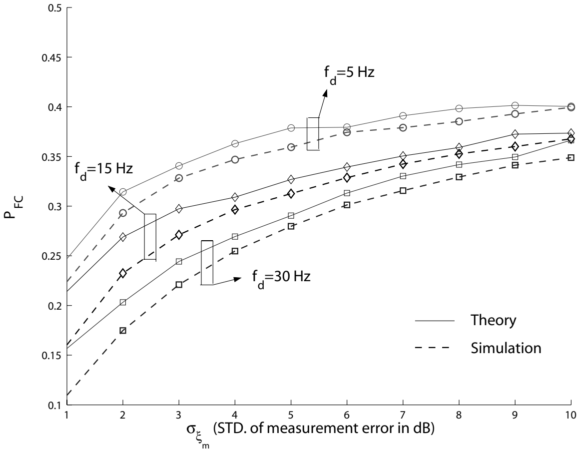

Fig. 2. The probability of false power control command (P_{FC}) subject to measurement errors for different Doppler frequencies, where $f_d = 30, 15$, and 5 Hz, and target $E_b/N_o = 5$ dB.

Now we discuss the statistical properties of ξ_m and ξ_p. As mentioned in [11], the measurement error is usually modelled as being log-normally distributed. Furthermore, the power control error can also be modelled as a log-normal random variable [12]. Since measurement errors are mainly influenced by the measurement scheme rather than the power control method, we assume that the measurement error ξ_m is independent of power control error ξ_p at the instant of issuing a power control command. Since ξ_m and ξ_p are two independent normal random variables with zero mean and standard deviation of σ_{ξ_m} and σ_{ξ_p}, respectively, the joint probability density function of ξ_p and ξ_m can be expressed as

$$f_{\xi_p\xi_m}(\xi_p, \xi_m) = \frac{1}{2\pi\sigma_{\xi_p}\sigma_{\xi_m}} \exp\left(\frac{-\varepsilon_p^2}{2\sigma_{\xi_p}^2}\right) \cdot \exp\left(\frac{-\varepsilon_m^2}{2\sigma_{\xi_m}^2}\right). \tag{7}$$

Because ξ_p and ξ_m are symmetric to their mean values and their means are zeros, we have

$$P(\xi_p + \xi_m < 0, \xi_p > 0) = P(\xi_p + \xi_m > 0, \xi_p \leq 0) \ . \tag{8}$$

Then, by substituting (7) and (8) into (6) , P_{FC} can be simplified as

$$\begin{aligned} P_{FC} &= 2P(\xi_p + \xi_m < 0, \xi_p > 0) \\ &= 2P(\xi_m < -\xi_p, \xi_p > 0) \\ &= \frac{1}{2} - \frac{\tan^{-1} \frac{\sigma_{\xi_p}}{\sigma_{\xi_m}}}{\pi} \ . \end{aligned} \tag{9}$$

From (9), we can easily calculate the probability of false power control command in terms of σ_{ξ_p} and σ_{ξ_m}.

IV. NUMERICAL RESULTS

In this section, we evaluate the uplink performance of the closed-loop power controlled CDMA system subject to measurement errors under the flat Rayleigh fading channel. We assume that the measurement errors are log-normal random variables [11]. Other parameters used in the simulations are listed in Table I.

Fig. 2 shows the impact of measurement errors on the the probability of false power control command (P_{FC}) for

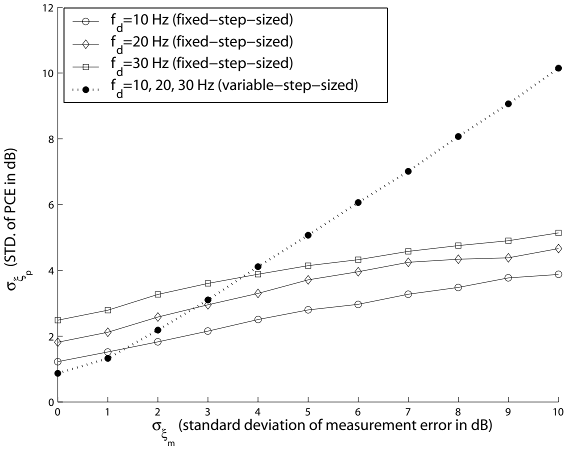

Fig. 3. Comparison of the fixed-step-sized closed-loop power control and the variable-step-sized closed-loop power control in terms of the correlation of power control errors and measurement errors.

TABLE I

SIMULATION PARAMETERS

Spreading Factor	4
Doppler Frequency	5-30 Hz
Power Control Period	0.667 msec
Power Control Step Size	1 dB
Target Eb/No	5 dB
Modulation Scheme	BPSK

different Doppler frequencies. One can find that the higher the Doppler frequency, the lower the value of P_{FC}. This phenomenon can be explained as follows. Because the radio channel changes faster at a higher Doppler frequency, the closed-loop power control mechanism may not be able to follow the fast varying channel with high Doppler frequency, thereby yielding a larger power control error. Due to a larger gap between the target SIR and the actual SIR, it is less likely to reverse the power control command from "power up" to "power down" or vice versa. In other words, a larger PCE (ξ_p in (6)) due to higher Doppler frequency can tolerate larger measurement errors (ξ_m in (6)), thereby resulting in a smaller P_{FC} as illustrated in the figure. The accuracy of simulation results in Fig. 2 is also validated by the analytical results obtained from (9). As shown, the simulation results in terms of P_{FC} are close to the analytical results derived from (9).

Fig. 3 shows the sensitivity of power control errors against measurement errors for the fixed-step-sized and the variable-step-sized closed-loop power control schemes. In our simulations, the variable-step-sized power control has higher resolution in power adaptations, i.e. with more bits to represent a power control command. Unlike the variable-step-sized power control having a strong correlation between measurement errors and power control errors, the fixed-step-sized CLPC power control errors increases less slowly as the standard deviation of measurement errors increases.

V. CONCLUSIONS

In this letter we have presented the analytical expression for the probability of false power control command (P_{FC}) in the closed-loop power control (CLPC) subject to measurement errors. It is found that the larger the Doppler frequency, the smaller the P_{FC}. Interestingly, as compared to the variable-step-sized power control, the fixed-step-sized CLPC is more robust to measurement errors due to its non-linear operation in the up/down control scheme.

The proposed analytical approach can be easily applied to analyze the performance of the closed-loop power control with different SIR measurement schemes. Thus, some interesting future research topics extended from this work include: (1) apply the proposed analytical framework to incorporate different SIR measurement schemes to evaluate the performance of the closed-loop power control in CDMA systems; (2) take into account of the impact of false power control command in performance analysis, for example, the BER analysis.

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